

CHAPTER 1

The first part of the book is devoted to the study of the basic concepts of the theory of groups. In this chapter we shall discuss the definition of a group, the properties of groups, and the construction of quotient groups.

Let G be a set and \cdot a binary operation on G . We say that (G, \cdot) is a group if the following conditions are satisfied: (1) \cdot is associative, (2) there is an identity element e in G such that $e \cdot a = a \cdot e = a$ for all a in G , and (3) for each a in G there is an inverse element a^{-1} in G such that $a \cdot a^{-1} = a^{-1} \cdot a = e$. The identity element e is unique, and the inverse element a^{-1} is unique. If $a \cdot b = b \cdot a$ for all a, b in G , then (G, \cdot) is called an abelian group. The order of a group G is the number of elements in G . A group G is finite if its order is finite, and infinite otherwise. The trivial group is the group consisting of the identity element e alone. A subgroup H of a group G is a subset of G which is itself a group under the same operation. The order of a subgroup H is the number of elements in H . Lagrange's theorem states that the order of a subgroup H of a finite group G divides the order of G . The quotient group G/H is the set of cosets of H in G , and the operation on G/H is defined by $(aH) \cdot (bH) = (a \cdot b)H$. The quotient group G/H is a group, and its order is $|G|/|H|$.

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